AQA

A-level

Mathematics

MFP4 – Further Pure 4

Mark scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\left(\overrightarrow{AB}=\right)\begin{pmatrix}2\\4\\-3\end{pmatrix}$			
	$\left(\overrightarrow{AC}=\right)\begin{pmatrix}1\\1\\-4\end{pmatrix}$	**B1**		Either $\overrightarrow{AB}$ or $\overrightarrow{AC}$ correct
	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\mathbf{i} & 2 & 1\\ \mathbf{j} & 4 & 1\\ \mathbf{k} & -3 & -4\end{vmatrix}=\begin{pmatrix}-13\\5\\-2\end{pmatrix}$	**M1** **A1**	 **3**	Vector product – two components correct (unsimplified) **CSO**
(b)	Area = $\frac{1}{2}\sqrt{13^2+5^2+2^2}$	**M1**		Correct use of formula for area of a triangle with their vector
	$=\frac{3}{2}\sqrt{22}$	**A1**	**2**	Accept any exact equivalents eg $\frac{1}{2}\sqrt{198}$ **CAO**
	Total		**5**	

Q2	Solution	Mark	Total	Comment
	det $(\mathbf{AB})^{-1}=-16$	**B1**		Correct evaluation of $\det(\mathbf{AB})^{-1}$ or det **(AB)** = $-\frac{1}{16}$
	Either use of det **A** x det **B** = det (**AB**) or det $\mathbf{B}^{-1}$ x det $\mathbf{A}^{-1}$ = det $(\mathbf{AB})^{-1}$			
	$\frac{1}{2}\times\det\mathbf{B}^{-1}=-16$	**M1**		Correct use of determinant rules and substituting their values
	$\det\mathbf{B}=-\frac{1}{32}$	**A1**	**3**	**CAO**
	Total		**3**	

Q3	Solution	Mark	Total	Comment
(a)(i)	r_1 replaced by $r_1 - r_2$ gives $\begin{vmatrix} a-4 & -12+3a & -a+4 \\ 4 & -3a & a-3 \\ -3 & 4a & a+4 \end{vmatrix}$	M1		Correct use of a row operation with at most one error in one term.
	$(a-4)\begin{vmatrix} 1 & 3 & -1 \\ 4 & -3a & a-3 \\ -3 & 4a & a+4 \end{vmatrix}$	A1	2	Or r_2 replaced by $r_2 - r_1$ give $(a-4)\begin{vmatrix} a & -12 & 1 \\ -1 & -3 & 1 \\ -3 & 4a & a+4 \end{vmatrix}$ **CAO**
(ii)	c_3 replaced by $c_3 + c_1$ gives $(a-4)\begin{vmatrix} 1 & 3 & 0 \\ 4 & -3a & a+1 \\ -3 & 4a & a+1 \end{vmatrix}$	M1		Combining rows or columns sensibly working towards a second linear factor, to a point where a factor can be extracted.
	$(a-4)(a+1)\begin{vmatrix} 1 & 3 & 0 \\ 4 & -3a & 1 \\ -3 & 4a & 1 \end{vmatrix}$	A1		Correct extraction of second linear factor and resulting determinant
		m1		Correct expansion of **their** resulting determinant to find final factor
	$-7(a-4)(a+1)(a+3)$	A1	4	Fully correct – must pull out the "7" for final **A1**. **CSO**
(b)	$-7(a-4)(a+1)(a+3)=0$	M1		Set their answer to **(a)(ii)** equal to 0
	$a=4, -3, -1$	A1	2	All three values obtained **CSO** – must have scored **4 marks** in **(a)(ii)** **SC** – if 3 in (a)(ii) due to "7" not pulled out can get **M1A1**.
(a)(ii)	**ALTERNATIVE** By direct expansion $\Delta = \begin{vmatrix} a & -12 & 1 \\ 4 & -3a & a-3 \\ -3 & 4a & a+4 \end{vmatrix}$ $= a\begin{vmatrix} -3a & a-3 \\ 4a & a+4 \end{vmatrix} + 12\begin{vmatrix} 4 & a-3 \\ -3 & a+4 \end{vmatrix} + \begin{vmatrix} 4 & -3a \\ -3 & 4a \end{vmatrix}$			
	$= -7a^3 + 91a + 84$	(M1)		Correct cubic obtained
	$= -7(a-4)(a^2+4a+3)$	(A1)		Correct quadratic factor seen with one linear factor identified
	$= -7(a-4)(a+3)(a+1)$	(A1)		Two linear factors correct
		(A1)	(4)	Fully correct **CSO**
	Total		**8**	

Q4	Solution	Mark	Total	Comment
(a)	$\begin{vmatrix} 5 & 2 & 4 \\ 7 & 4 & 6 \\ 6 & k-2 & k \end{vmatrix}$			
	$=5\begin{vmatrix} 4 & 6 \\ k-2 & k \end{vmatrix} - 7\begin{vmatrix} 2 & 4 \\ k-2 & k \end{vmatrix} + 6\begin{vmatrix} 2 & 4 \\ 4 & 6 \end{vmatrix}$	M1		Correct row or column expansion
	$=5(-2k+12)-7(-2k+8)+6(-4)$	m1		Correct 2 by 2 determinant expansion
	$= 4k-20$ $4k-20 \neq 0$ $k \neq 5$	A1	3	Fully correct linear expression and correct conclusion – **AG** – **CSO**
	ALTERNATIVE When k = 5, det **A** = $\begin{vmatrix} 5 & 2 & 4 \\ 7 & 4 & 6 \\ 6 & 3 & 5 \end{vmatrix}$ Then row 1 + row 2 = 2 × row 3 so det **A** = 0 since **A** is non-singular then $k \neq 5$	(SC1)	(1)	Correct substitution of k = 5 and an attempt at row operations or evaluation of determinant Correct justification/evaluation of det **A** = 0 Fully correct conclusion
(b)	$\begin{bmatrix} 12-2k & 36-7k & 7k-38 \\ 2k-8 & 5k-24 & 22-5k \\ -4 & -2 & 6 \end{bmatrix}$	M1 A(2,1,0)		**M1** cofactor matrix – one full row or column correct **A1** – at least six entries correct **A2** – all entries correct
	$\mathbf{A}^{-1} = \dfrac{1}{4k-20}\begin{bmatrix} 12-2k & 2k-8 & -4 \\ 36-7k & 5k-24 & -2 \\ 7k-38 & 22-5k & 6 \end{bmatrix}$	m1 A1	5	Divide by their determinant **and** transpose their matrix – must have first **M1** **CAO**

(c)	Comparing gives $k = -1$, hence $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-24}\begin{bmatrix} 14 & -10 & -4 \\ 43 & -29 & -2 \\ -45 & 27 & 6 \end{bmatrix}\begin{bmatrix} a \\ -a \\ -5a \end{bmatrix}$			Must be evidence of using their $\mathbf{A}^{-1}$ from **part (b)**
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-24}\begin{bmatrix} 14a+10a+20a \\ 43a+29a+10a \\ -45a-27a-30a \end{bmatrix}$	**M1** **A1F**		**M1** – Correct identification of $k = -1$ and attempt at $\mathbf{A}^{-1}$ **v** with at least one of their components correct – can be unsimplified, but with no k's. **A1F** Two of their components correct – can be unsimplified. Follow through their $\mathbf{A}^{-1}$ (Condone wrong or missing 1/|**A**| for **M1A1F**)
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-24}\begin{bmatrix} 44a \\ 82a \\ -102a \end{bmatrix}$	**A1**		All three components correct – terms collected but need not be fully cancelled
	$x = -\frac{11a}{6}$ $y = -\frac{41a}{12}$ $z = \frac{17a}{4}$	**A1**	**4**	Correct and fully simplified , with variables clearly identified – **CSO** Any method with does not use $\mathbf{A}^{-1}$ **v** scores zero marks **NMS** = 0 marks
	Total		**12**	

Q5	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix}1 & 1 & 2\\ 0 & 5 & -1\\ 2 & p & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=2\begin{bmatrix}x\\ y\\ z\end{bmatrix}$			
	Hence (1) $x+y+2z=2x$ (2) $5y-z=2y$ (3) $2x+py+z=2z$	**M1**		Use of **Mv** = 2**v** to obtain **three** correct equations
	(2) simplifies as $z=3y$ Using this with (1) gives $x=7y$	**A1**		Correctly obtaining each variable in terms of a single variable/parameter
	Substituting in (3) gives $14y+py-3y=0$	**m1**		Substituting in final equation to obtain an equation in *p* and one other variable
	Hence $p=-11$	**A1**	**4**	**CSO**
	ALTERNATIVE Substituting $\lambda=2$ $\begin{vmatrix}-1 & 1 & 2\\ 0 & 3 & -1\\ 2 & p & -1\end{vmatrix}=0$	**(M1)**		Fully correct **equation** with $\lambda=2$ substituted in.
	$-1\begin{vmatrix}3 & -1\\ p & -1\end{vmatrix}+2\begin{vmatrix}1 & 2\\ 3 & -1\end{vmatrix}=0$	**(A1)**		Correct expansion by row or column of 3 by 3 determinant
	Hence $3-p-14=0$	**(m1)**		Correct expansion of 2 by 2 determinants to form linear equation
	giving $p=-11$	**(A1)**	**(4)**	Correct value found **- CSO**
(b)	$\begin{vmatrix}1-\lambda & 1 & 2\\ 0 & 5-\lambda & -1\\ 2 & -11 & 1-\lambda\end{vmatrix}(=0)$			
	$(1-\lambda)\begin{vmatrix}5-\lambda & -1\\ -11 & 1-\lambda\end{vmatrix}+2\begin{vmatrix}1 & 2\\ 5-\lambda & -1\end{vmatrix}(=0)$	**M1**		Correct row/column expansion of their $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert(=0)$, may be in terms of *p*.
	$(1-\lambda)[(5-\lambda)(1-\lambda)-11]+2(-1-10+2\lambda)(=0)$	**m1**		Correct expansion of their 2 by 2 determinants – dependent on first **M1**, may be in terms of *p*.
	$\lambda^3-7\lambda^2-4\lambda+28(=0)$	**A1**		Correct expanded characteristic equation
	$(\lambda-2)(\lambda+2)(\lambda-7)(=0)$	**A1**		Correct factors
	$(\lambda_1=2),\lambda_2=-2,\lambda_3=7$	**A1**	**5**	Correct eigenvalues obtained **and** identified

(c)	Any eigenvector found $\begin{bmatrix}7\\1\\3\end{bmatrix}$ $\begin{bmatrix}-5\\1\\7\end{bmatrix}$ $\begin{bmatrix}-1\\2\\-4\end{bmatrix}$	**M1**		Obtaining a correct eigenvector
	Hence equation of line is $\mathbf{r}=\begin{bmatrix}7\\1\\3\end{bmatrix}t$	**A1**	**2**	Any fully correct vector format Must have "$\mathbf{r}=\cdots$ "
	Total		**11**	

Q6	Solution	Mark	Total	Comment
(a)	All points on the **line** remain in the **same position** (after transformation)	E1	1	Must explain what "invariant" means.
(b)	T maps (x, y) to (x, y) $x = -2x + 2y + 4$	M1		Replace **either** *x'* by *x* or *y'* by *y*.
	$y = 3x - y - 4$	A1		Replace **both** *x'* & *y'* by *x* & *y*
	Any correct simplified form e.g. $0 = 3x - 2y - 4$ (is the line of invariant points).	A1	3	Correct line identified – **ACF** (e.g. $y = \frac{3}{2}x - 2$)
(c)	Using $y = mx + c$ gives $x' = -2x + 2(mx + c) + 4$ $y' = 3x - (mx + c) - 4$	B1		Correct substitution of $y = mx + c$
	Then using $y' = mx' + c$ gives $3x - y - 4 = m(-2x + 2y + 4) + c$	M1		Substitution of $y' = mx' + c$
	$3x - (mx + c) - 4 = m[-2x + 2(mx + c) + 4] + c$ $0 = (2m^2 - m - 3)x + (2mc + 4m + 2c + 4)$	A1		Fully correct simplification – collecting appropriately
	$0 = (2m - 3)(m + 1)x + 2(m + 1)(c + 2)$ Hence when $m = -1$, c can be any real value	m1		Factorising and solving to find *m*, clear reference to putting $m = -1$ into the other eq.
	So $y = -x + c$ are the infinite set of invariant lines **[NB** $m = \frac{3}{2}, c = -2$ gives line of invariant points]	A1	5	Fully correct conclusion **– CSO**
	Total		9	

Q7	Solution	Mark	Total	Comment
(a)	Determinant of **M** = $8k^3$	**B1**	**1**	Correct evaluation of det **M**
(b)(i)	Volume scale factor = $\dfrac{0.75}{48}=\dfrac{1}{64}$			
	Hence $8k^3=\dfrac{1}{64}$	**M1**		Forms a correct equation in *k* **PI** correct *k* or Scale Factor
	so $k=\dfrac{1}{8}$ and	**A1**		*k* value correct
	enlargement scale factor = $\dfrac{1}{4}$	**A1**	**3**	Scale factor correct
(ii)	Let **N** = required matrix, hence			
	$\mathbf{N}\begin{pmatrix}2k & 0 & 0\\ 0 & 2k & 0\\ 0 & 0 & 2k\end{pmatrix}=\begin{pmatrix}-k\sqrt{3} & 0 & -k\\ 0 & 2k & 0\\ k & 0 & -k\sqrt{3}\end{pmatrix}$	**M1**		Combines appropriate matrices to find required matrix associated with $\mathbf{T_2}$ – *k* value may be substituted.
	N = $\begin{bmatrix}\frac{-\sqrt{3}}{2} & 0 & -\frac{1}{2}\\ 0 & 1 & 0\\ \frac{1}{2} & 0 & \frac{-\sqrt{3}}{2}\end{bmatrix}$	**A1**		Correct matrix obtained.
	T_2 is a rotation about the *y* axis	**M1**		And no other transformation given.
	$\cos\theta=-\dfrac{\sqrt{3}}{2}$ and $\sin\theta=-\dfrac{1}{2}$	**m1**		**PI** by 210^0 **OE**
	210^0	**A1**	**5**	Correct angle found – also accept -150^0 (any equivalences accepted)
	Total		**9**	

Q8	Solution	Mark	Total	Comment
(a)	$\mathbf{n} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$			
	$c = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} . \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = 5(3) + (2)(-1) + (-1)(2)$	**M1**		Correct use of scalar product with correct **n** and correct point to find a value of c
	$\mathbf{r}.\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = 11$	**A1**	**2**	Correct value of c and use of given format
(b)	$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix}$ or $\mathbf{v}_2 = \begin{bmatrix} -2 \\ -4 \\ 0 \end{bmatrix}$ or $\mathbf{v}_3 = \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix}$	**B1**		Any correct direction vector
	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & 2 & -2 \\ \mathbf{j} & -2 & -4 \\ \mathbf{k} & -3 & 0 \end{vmatrix} = \begin{bmatrix} -12 \\ 6 \\ -12 \end{bmatrix}$	**M1** **A1**		Using vector product of two correct direction vectors
	Hence $-12x + 6y - 12z = p$ $p = -30$	**m1**		Attempts to use their **n** and correct point to find their value of p
	$2x - y + 2z = 5$	**A1**	**5**	$d = 5$ **CSO**
(c)	$\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} . \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = 6$	**B1**		Correct evaluation of scalar product
	$\sqrt{5^2 + 2^2 + 1^2} = \sqrt{30}$ $\sqrt{2^2 + 1^2 + 2^2} = 3$			
	$\cos\theta = \frac{6}{3\sqrt{30}}$	**M1**		Use of correct normals with $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$
	$(= \frac{6\sqrt{30}}{90}) = \frac{\sqrt{30}}{15}$	**A1**	**3**	**AG** – Be convinced

	ALTERNATIVE			
	$\left\| \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \right\| = \sqrt{234}$	**(B1)**		Correct evaluation of vector product. **Must** have modulus
	$\sin\theta = \dfrac{\sqrt{234}}{3\sqrt{30}}$	**(M1)**		Use of correct normals with $\sin\theta = \dfrac{\|\mathbf{a}\times\mathbf{b}\|}{\|\mathbf{a}\|\|\mathbf{b}\|}$
	$\cos^2\theta = 1-\sin^2\theta = 1-\dfrac{26}{30} = \dfrac{4}{30}$			
	$\therefore \cos\theta = \dfrac{\sqrt{30}}{15}$	**(A1)**	**(3)**	**AG – Must** use $\cos^2\theta+\sin^2\theta=1$ else **A0**
(d)(i)	$\begin{vmatrix} 5 & 2 & -1 \\ 2 & -1 & 2 \\ 4k-8 & k+1 & -4k^2 \end{vmatrix} = 0$			
	$(4k-8)\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} - (k+1)\begin{vmatrix} 5 & -1 \\ 2 & 2 \end{vmatrix} - 4k^2\begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = 0$	**M1**		Correct expansion by row or column. (Substituting k=1 scores **M0**).
	$(4k-8)(3)-(k+1)(12)-4k^2(-9)=0$	**A1**		Correct expansion of 2 by 2 determinants
	$36k^2-36=0$ $36(k-1)(k+1)=0$	**m1**		Valid method for solving a quadratic – dependent on first **M1**
	$k=1,-1$	**A1**	**4**	Both values correct – **AG** for $k=1$
(ii)	k =1 results in the equation for Π_3 as $-4x+2y-4z=8$. Or valid method for comparing normals of Π_3 and Π_2	**M1**		Substituting k =1 to get correct equation for Π_3.
	Π_3 and Π_2 are parallel (Π_1 is not parallel).	**A1**		Must identify correct parallel planes.
	k = –1 results in the equation for Π_3 as $-12x-4z=8$.	**M1**		Substituting k = –1 to get correct equation for Π_3. Must have scored **4 marks** in (d)(i).
	Since planes have **no common point**. The three planes form a **prism**.	**A1**	**4**	Fully correct deduction
	Total		**18**	
	TOTAL		**75**	